The Economics of Contingent Re-Auctions

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Introduction

- Motivated by design of FCC Spectrum License Auction 73 "700 MHz"
  - started January 24, 2008 and ended March 18, 2008
  - 1,099 spectrum licenses
  - 62 MHz nationwide in the 698–806 MHz band
  - good propagation characteristics, unencumbered
- Format: "contingent re-auction" or "do-over auction"
  - Offer licenses with restrictions and a commitment to immediately re-auction them without the restrictions if the reserve price is not met
  - FCC believes the restrictions are in the public interest, but does not want to sacrifice too much revenue in the process of imposing them

Introduction – FCC's contingent re-auction

- A-block, B-block, E-block
  - 12 MHz EA licenses, 12 MHz CMA licenses, 6 MHz EA licenses
  - reserves: $1.81 billion, $1.38 billion, $0.90 billion
  - restrictions:
    - geographic benchmarks for coverage (35% in 4 years, 70% in 10 years)
- C-block
  - 22 MHz REAG licenses
  - reserve price $4.64 billion
  - restrictions:
    - population benchmarks for coverage (40% in 4 years, 75% in 10 years)
    - open platforms for devices and applications (no locking and no blocking)
- These reserve prices were met, so licenses were allocated with restrictions

Introduction – Alitalia example

- Sale of Italian airline Alitalia
  - offered for sale with a series of restrictions (e.g., limitations on the ability to fire employees)
  - after receiving no attractive bids, offered for sale again with fewer restrictions via bilateral private negotiations
  - still unsold

Introduction – General problem

- Environment with seller-benefitting restrictions
  - seller has one object for sale
  - seller can 'damage' the object, for example by restricting its use
  - if the restricted object is sold, the seller receives a benefit B in addition to the sale price
  - if the unrestricted object is sold, the seller receives no benefit beyond the sale price
  - bidders value the unrestricted object more than the restricted object and have private values
- Efficiency requires that the object be restricted if and only if B is larger than the difference between the highest value for the unrestricted object and the highest value for the restricted object
- Questions:
  - analysis of the contingent re-auction
  - design of an efficient mechanism (max expected total surplus)
  - design of an optimal mechanism (max expected seller surplus)

Related literature

Mechanism design

- Mechanism design with one-dimensional types
  - Large literature
  - Optimal auction design – Myerson (1981)
- Mechanism design with multi-dimensional types
  - Rochet (1985), Matthews & Moore (1987), McAfee & McMillan (1988), Armstrong (1996)
  - Rochet & Choné (1998), Manelli & Vincent (2006, 2007)

Related literature

- Auctions with resale by seller
  - Horstmann & LaCasse (1997), Cassady (1967), Ashenfelter (1989), Porter (1995), McAfee & Vincent (1997)
- Auctions with resale by buyer
  - Gupta & Lebrun (1999), Haile (2000, 2001, 2003), Zheng (2002), Garratt & Tröger (2005), Hafalir & Krishna (2006), Garratt, Tröger, & Zheng (2006), Lebrun (2007), Pagnozzi (2007)
- Monopolist chooses product quality
  - Mussa & Rosen (1978), Deneckere & McAfee (1996)

Outline of talk

- Model
- Equilibria of the contingent re-auction
- Efficient mechanism
- Optimal mechanism
- Comparisons
- Conclusions

Model

- Single object that can be restricted
- Benefit $B > 0$ for the seller if sold restricted
- n bidders
- Bidder i has type $\theta_i = (l_i, h_i)$ drawn independently from $F_i(l, h)$, with support

$$\left\{(l, h) \in \mathbb{R}^2_+ \mid l \in [\underline{l}, \bar{l}],\ h \in [\underline{h}, \bar{h}],\ l \leq h\right\}$$

- Private values: l_i for restricted and h_i for unrestricted object

$$\begin{cases} \text{surplus if win restricted:} & l_i - \text{amount paid} \\ \text{surplus if win unrestricted:} & h_i - \text{amount paid} \end{cases}$$

Model of the contingent re-auction

1. The seller offers the restricted object in a second-price or ascending-bid auction with reserve price r
2. If the reserve is not met, the seller offers the unrestricted object in a second-price or ascending-bid auction with no reserve price

- Can allow a positive reserve price at the second auction
- Results for the contingent re-auction assume symmetry ($F_i = F$)

Equilibrium of the contingent re-auction

Proposition (Existence)

A perfect Bayesian equilibrium of the contingent re-auction exists.

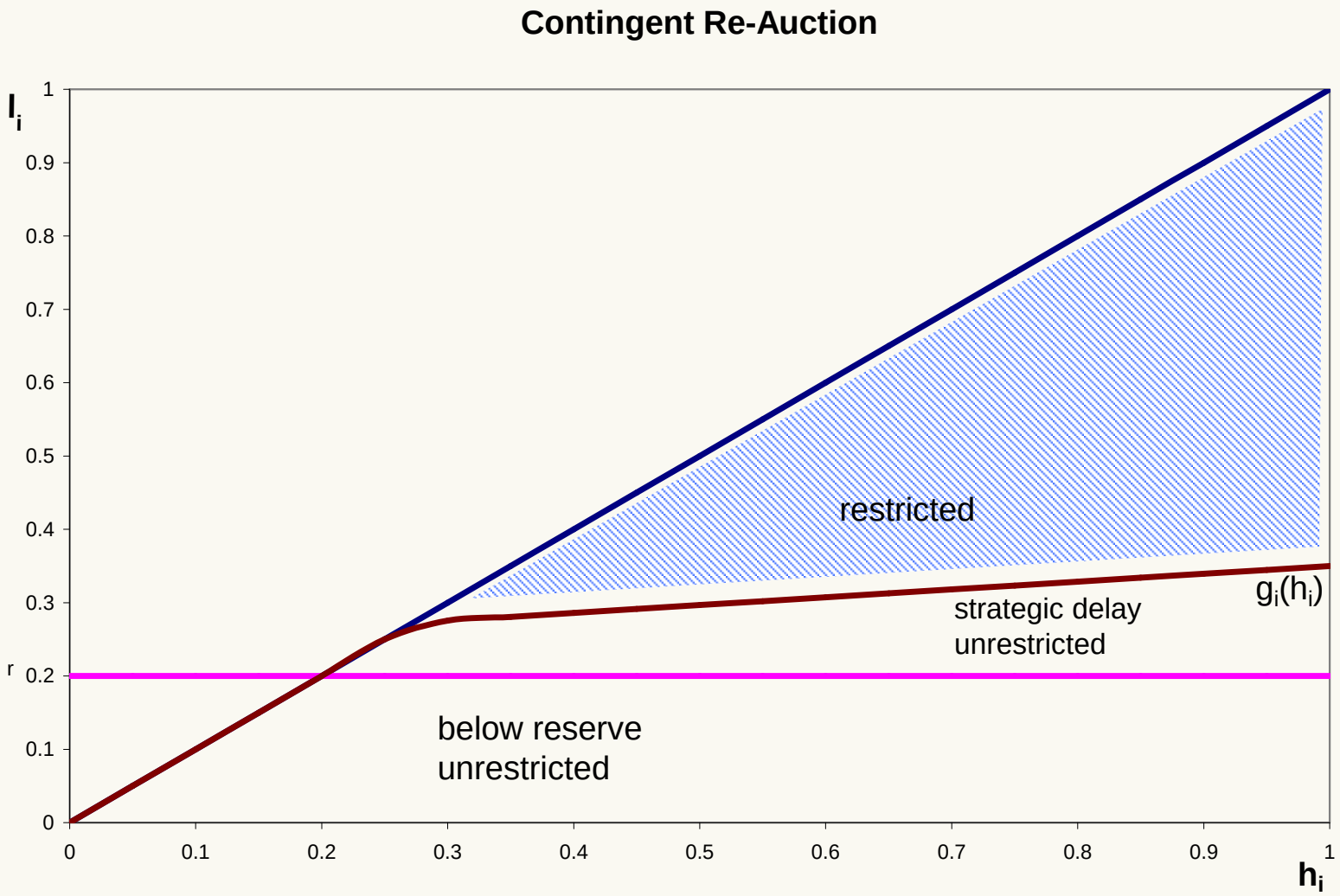
- In the second auction, bidders bid truthfully
- In the first auction, bidders either bid truthfully or bid zero
  - each bidder has a threshold function $g_i(h_i) \geq r$ such that

$$\begin{cases} \text{if } l_i > g_i(h_i), & \text{bid (up to) } l_i \text{ in first auction} \\ \text{if } l_i < g_i(h_i), & \text{do not bid in first auction} \end{cases}$$

Proposition (Characterization)

In any perfect Bayesian equilibrium in undominated strategies, for each bidder i there is a nondecreasing function $g_i : [\underline{h}, \bar{h}] \longrightarrow [r, \bar{l}]$ such that i's bid in the first auction is zero if $l_i < g_i(h_i)$ and l_i otherwise.

Allocation in the contingent re-auction



Delay in the contingent re-auction

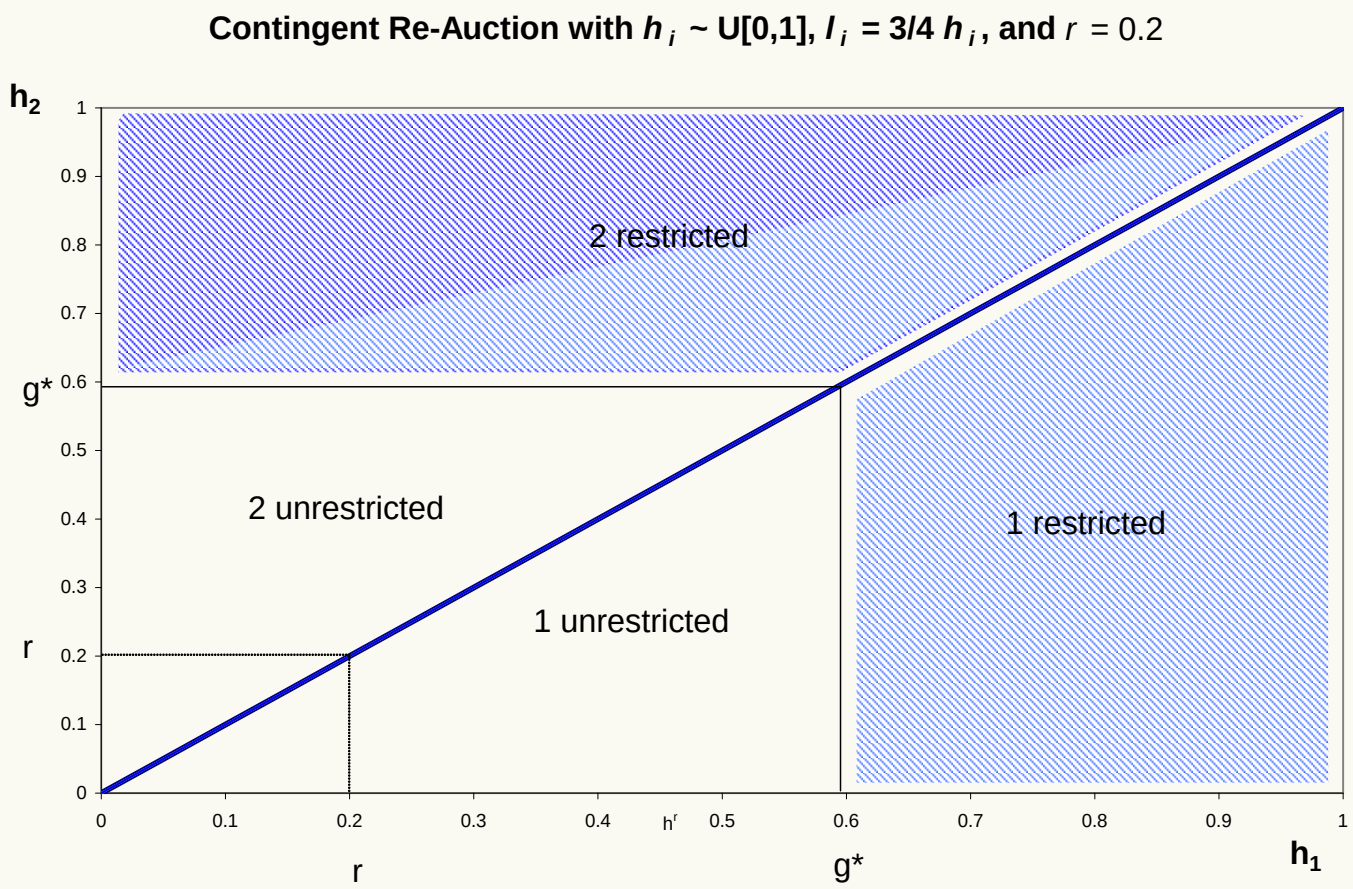
- There is no equilibrium in which the bidders always bid truthfully (i.e., bid l_i whenever $l_i \geq r$) in the first auction
- In every equilibrium there is 'delay' in the sense that the second auction is reached with a probability strictly higher than $\Pr\left(\max_i l_i \leq r\right)$

Proposition (Equilibrium Delay)

If $r > \underline{l}$, in every equilibrium, there is an open set of types (l_i, h_i) with $l_i > r$ who bid zero in the first auction.

Evaluating the efficiency of the contingent re-auction

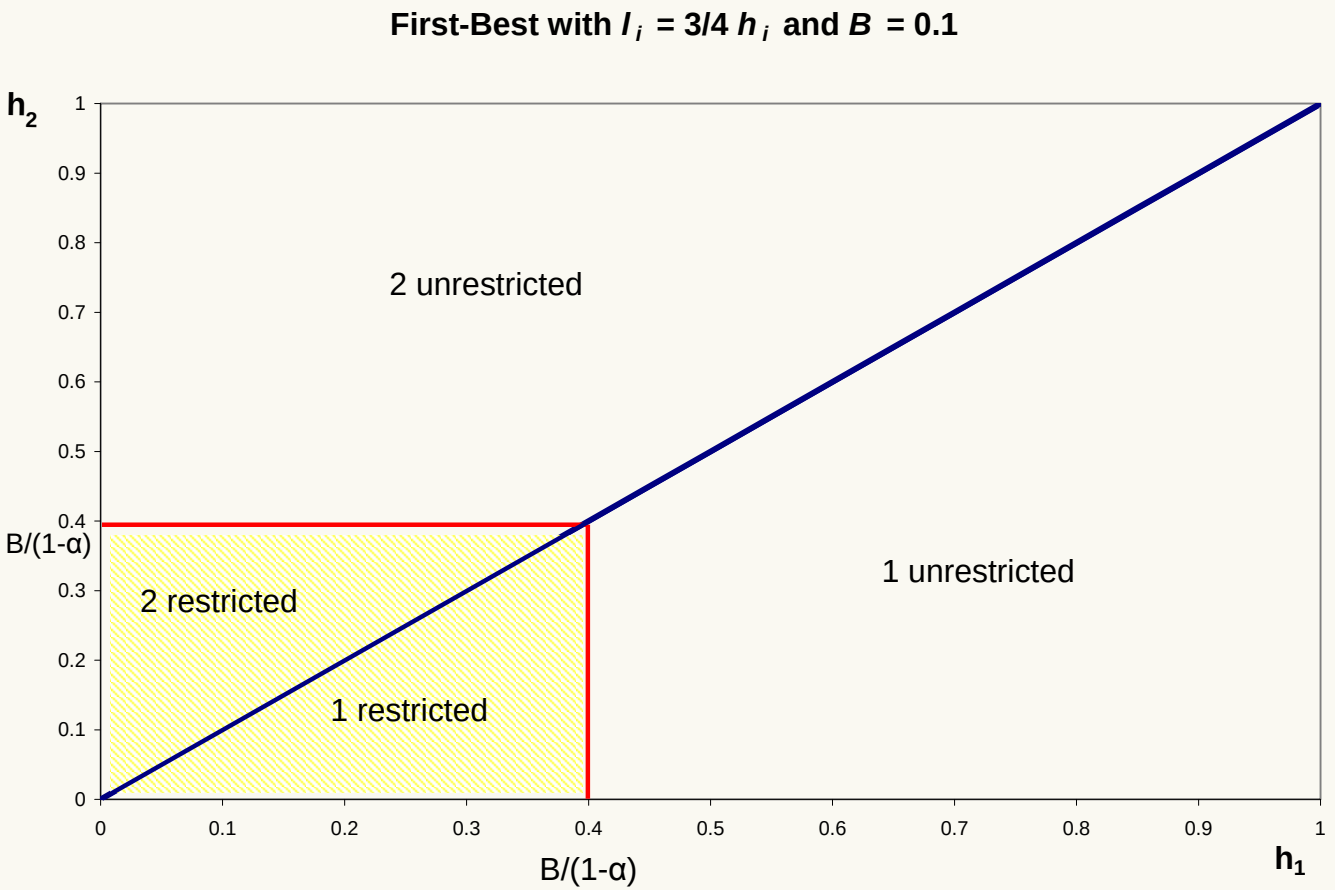
- Example: symmetric bidders with $l_i = \alpha h_i$ for $\alpha \in (0, 1)$
- Given r, there exists g^* such that bidder i bids zero in the first auction if $h_i < g^*$



First-best with one-dimensional types

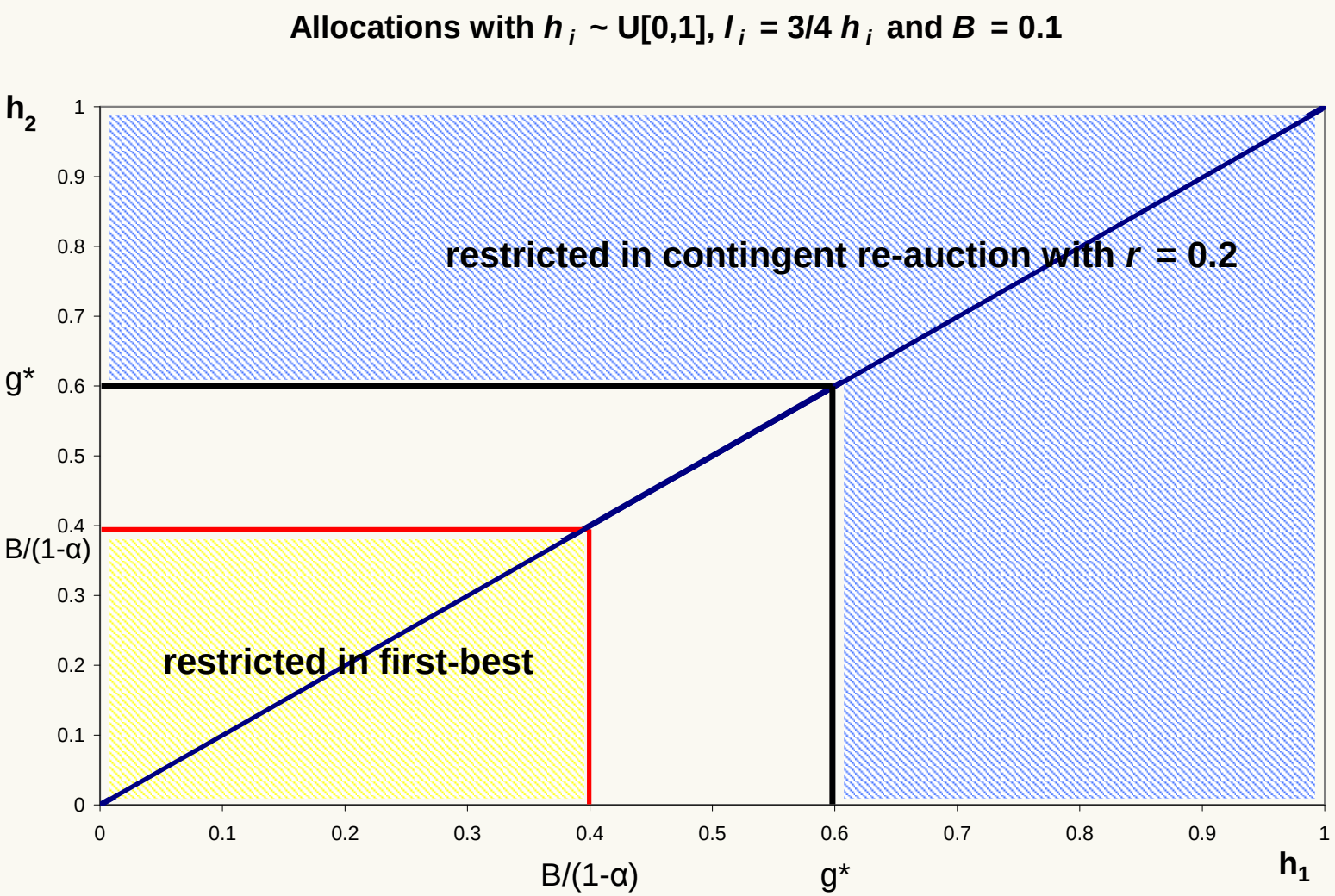
- Continue to assume $l_i = \alpha h_i$ for $\alpha \in (0,1)$
- Restrict the object if

$$B > \max_i h_i - \max_i l_i \quad \Leftrightarrow \quad \frac{B}{1-\alpha} > \max_i \{h_i\}$$



Inefficiency in the contingent re-auction with one-dimensional types

- The contingent re-auction can be highly inefficient in this environment



Return to multi-dimensional types

- What mechanism would be efficient?
- What mechanism would be optimal for the seller?

Defining the first-best (multi-dimensional types)

- Allocate restricted to i if

$$B + l_i > \max \left\{ \max_j h_j, \quad \max_{j \neq i} B + l_j \right\}$$

- Allocate unrestricted to i if

$$h_i > \max \left\{ \max_{j \neq i} h_j, \quad \max_j B + l_j \right\}$$

An efficient mechanism exists

Proposition (Efficient Mechanism)

The first-best outcome can be achieved with an exclusive buyer mechanism in which buyers first bid in a second-price or ascending-bid auction (no reserve) for the right to face the choice between purchasing the restricted object for an incremental payment of zero or the unrestricted object for an incremental payment of B.

- Bidder i bids $\max\{l_i, h_i - B\}$ at the initial auction
- The 'right' bidder wins and makes the 'right' choice
- Outcome equivalent (ex post) to a VCG mechanism, but indirect implementation (maintains privacy)
- Can be used with an arbitrary number of possible restrictions

Implementing the first-best

- The exclusive buyer mechanism should be straightforward for the FCC to implement
- For each license, the FCC would need to announce the extra payment for the unrestricted license (equivalently, a bidding credit for the restricted license)
  - currently announce a minimum opening bid and reserve price
- Run standard SMR (with or without package bidding) format to determine winner and initial payment
- Winner then chooses between restricted and unrestricted, with incremental payment for unrestricted (or credit for restricted)

Optimal mechanism (multi-dimensional types)

- What is the seller optimal mechanism?

- Exclusive buyer mechanism has one parameter, price P for unrestricted
- Setting P equal to B gives efficiency
- Price P can also be chosen to maximize seller surplus
- Can add a reserve price to the initial stage, so there are two parameters
- We show this mechanism can be optimal for the seller

Exclusive buyer mechanism can be optimal

Proposition (optimality with one-dim types)

The exclusive buyer mechanism that maximizes seller surplus is optimal if buyers are symmetric, $l_i = \alpha h_i$ for $\alpha \in (0,1)$, and $h - \frac{1-F(h)}{f(h)}$ is increasing in h.

Proposition (DS optimality with multi-dim types)

The exclusive buyer mechanism that maximizes seller surplus is optimal among dominant-strategy mechanisms if buyers are symmetric and F is uniform.

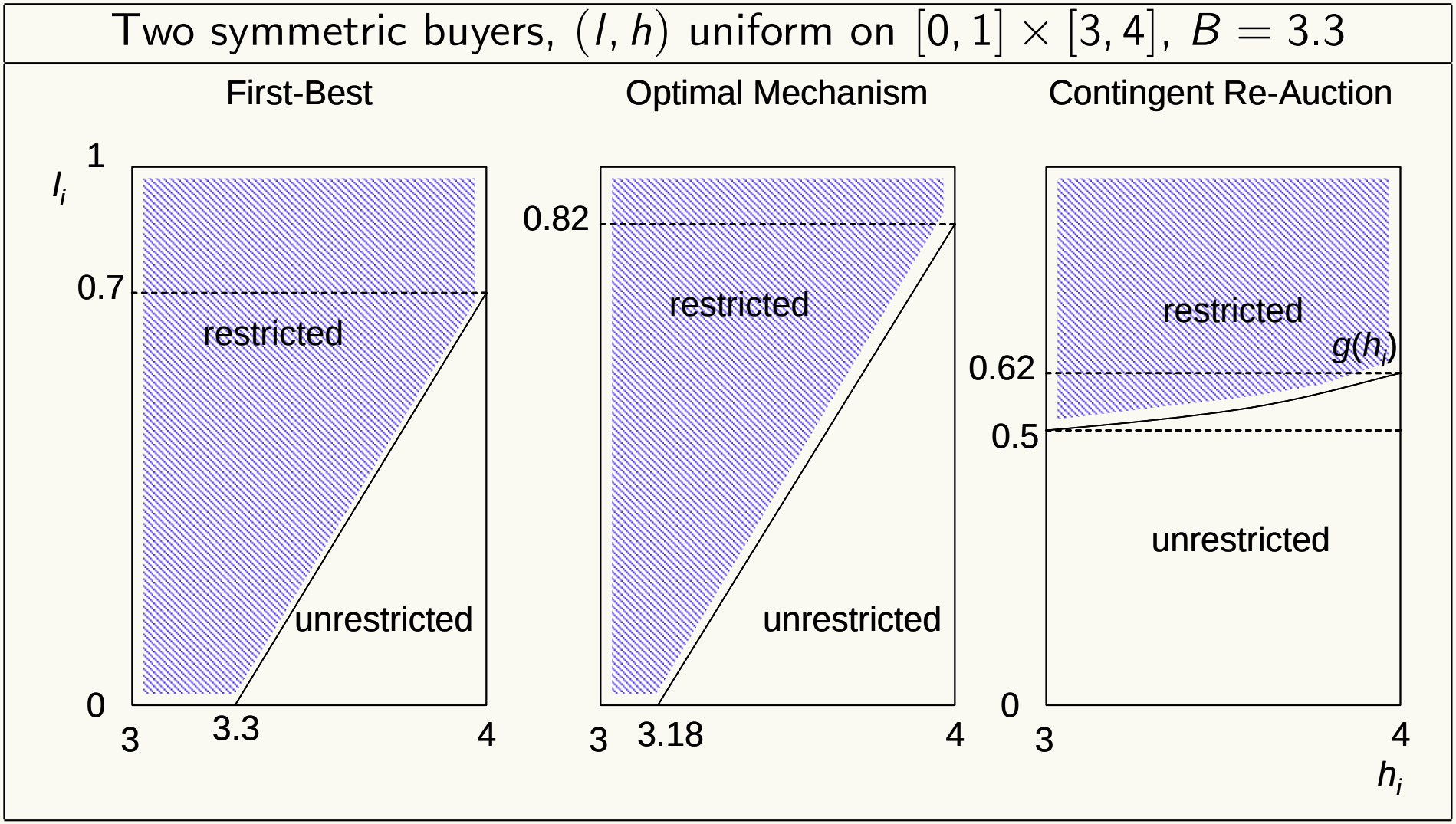
Conjecture

The exclusive buyer mechanism that maximizes seller surplus is optimal among dominant-strategy mechanisms.

Exclusive buyer mechanism

- Exclusive buyer mechanism performs well in a variety of environments
- Efficient when P equals B
- Optimal for seller when P is chosen to maximize seller surplus
  - at least among dominant-strategy mechanisms – loosely, a bidder would not want to change its strategy if a leak revealed information about opponents' strategies
  - may be optimal to add a reserve price in the first stage
- Varying P between B and the seller-optimal price allows a balance of efficiency and seller surplus maximization

Comparison of allocations



Conclusion (1 of 2)

- Contingent re-auction
  - results offer guidance for setting the reserve price
  - can guarantee that the contingent re-auction is at least as good (in expectation) as a single auction for the unrestricted object with no reserve by setting r such that $r + B$ is equal to the expected revenue from selling the unrestricted object
- Other mechanisms
  - Efficient mechanism – may improve efficiency, seller surplus, and buyer surplus
  - Optimal mechanism – potentially large increases in seller surplus

Conclusion (2 of 2)

- Our primary recommendation is that in environments with seller-benefitting restrictions consider using an exclusive buyer mechanism, tailored to maximize efficiency or seller surplus depending upon the seller's objectives.
- Exclusive buyer mechanism in practice:
  - posting system used by Japan's Nippon Professional Baseball to offer Japanese players to Major League Baseball
  - MLB Office of the Commissioner holds an auction for the right to negotiate with the Japanese player
  - Recently, the Red Sox paid $51.11 million for the opportunity to negotiate with Daisuke Matsuzaka, who they then signed to a six-year, $52 million contract.